

MODULE – I

NUMBER SYSTEMS

Many number systems are in use in digital technology. The most common are the decimal, binary, octal, and hexadecimal systems. The decimal system is clearly the most familiar to us because it is a tool that we use every day. Examining some of its characteristics will help us to better understand the other systems. In the next few pages we shall introduce four numerical representation systems that are used in the digital system. There are other systems, which we will look at briefly.

- Decimal
- Binary
- Octal
- Hexadecimal

Decimal System

The decimal system is composed of 10 numerals or symbols. These 10 symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Using these symbols as digits of a number, we can express any quantity. The decimal system is also called the base-10 system because it has 10 digits.

Decimal Examples 3.14_{10} , 52_{10} , 1024_{10} , 64000_{10}

Binary System

In the binary system, there are only two symbols or possible digit values, 0 and 1. This base-2 system can be used to represent any quantity that can be represented in decimal or other base system. In digital systems the information that is being processed is usually presented in binary form. Binary quantities can be represented by any device that has only two operating states or possible conditions.

E.g. A switch is only open or closed. We arbitrarily (as we define them) let an open switch represent binary 0 and a closed switch represent binary 1. Thus we can represent any binary number by using series of switches.

Octal System

The octal number system has a base of eight, meaning that it has eight possible digits: 0, 1, 2, 3, 4, 5, 6, 7.

Octal to Decimal Conversion

$$237_8 = 2 \times (8^2) + 3 \times (8^1) + 7 \times (8^0) = 159_{10}$$

$$24.6_8 = 2 \times (8^1) + 4 \times (8^0) + 6 \times (8^{-1}) = 20.75_{10}$$

$$11.1_8 = 1 \times (8^1) + 1 \times (8^0) + 1 \times (8^{-1}) = 9.125_{10}$$

$$12.3_8 = 1 \times (8^1) + 2 \times (8^0) + 3 \times (8^{-1}) = 10.375_{10}$$

Hexadecimal System

The hexadecimal system uses base 16. Thus, it has 16 possible digit symbols. It uses the digits 0 through 9 plus the letters A, B, C, D, E, and F as the 16 digit symbols.

Hexadecimal to Decimal Conversion

$$24.6_{16} = 2 \times (16^1) + 4 \times (16^0) + 6 \times (16^{-1}) = 36.375_{10}$$

$$11.1_{16} = 1 \times (16^1) + 1 \times (16^0) + 1 \times (16^{-1}) = 17.0625_{10}$$

$$12.3_{16} = 1 \times (16^1) + 2 \times (16^0) + 3 \times (16^{-1}) = 18.1875_{10}$$

Code Conversion

Converting from one code form to another code form is called code conversion, like converting from binary to decimal or converting from hexadecimal to decimal.

Binary-To-Decimal Conversion

Any binary number can be converted to its decimal equivalent simply by summing together the weights of the various positions in the binary number which contain a 1. e.g.

$$11011_2 = 2^4 + 2^3 + 0_1 + 2_1 + 2_0 = 16 + 8 + 0 + 2 + 1 = 27_{10}$$

Octal-To-Binary Conversion

Each Octal digit is represented by three binary digits.

Example: $472_8 = (100)(111)(010)_2 = 100111010_2$

Octal-To-Hexadecimal Hexadecimal-To-Octal Conversion

- Convert Octal (Hexadecimal) to Binary first.
- Regroup the binary number by three bits per group starting from LSB if Octal is required.
- Regroup the binary number by four bits per group starting from LSB if Hexadecimal is required.

Arithmetic Operation using 1's and 2's Complement

1's Complement

The 1's complement of a binary number is defined as the value obtained by inverting all the bits in the binary representation of the number (swapping 0's for 1's and vice versa). The 1's complement of the number then behaves like the negative of the original number in some arithmetic operations.

Example: 1's complement of 10111 is 01000.

2's Complement

To get 2's complement of a binary number we add one (1) to the 1's complement of that same binary number.

Example: 2's complement of 10111 is; 1's complement of 10111 + 1

$$\Rightarrow 01000 + 1 = 01001$$

Arithmetic Operation

1. Using 1's complement:

Example1: Subtract 134 from 168.

$$168-134 = 168 + (-134)$$

Binary representation of 168= 1010 1000

Binary representation of 134= 1000 0110

Binary representation of -134= 0111 1001

[Because 1's complement represents the negative magnitude of a binary number]

$$168 + (-134) = 1010 1000$$

$$\begin{array}{r} + 0111 1001 \\ \hline \underline{10010 0001} \end{array}$$

As a carry bit is present, 1 will be added to the result and it represents that the result is positive.

$$0010 0001 + 1 = 0010 0010$$

Decimal representation of 0010 0010 is 34.

168-134=34; hence the result is correct.

Example2: Subtract 168 from 134.

$$134-168 = 134 + (-168)$$

Binary representation of 134= 1000 0110

Binary representation of 168= 1010 1000

Binary representation of -168= 0101 0111

[Because 1's complement represents the negative magnitude of a binary number]

$$\begin{array}{r}
 134 + (-168) = 1000\ 0110 \\
 + 0101\ 0111 \\
 \hline
 1101\ 1101
 \end{array}$$

As a carry bit is absent, 1's complement of this value will be the final result and absence of carry bit represents that the result is negative.

$$1's\ complement\ of\ 1101\ 1101 = 0010\ 0010$$

Decimal representation of 0010 0010 is 34. As carry bit is absent the result is negative i.e -34

134 -168= -34; hence the result is correct.

2. Using 2's complement:

Example 1: Subtract 96 from 118.

$$118-96 = 118 + (-96)$$

Binary representation of 118= 0111 0110

Binary representation of 96= 0110 0000

Here 2's complement represents the negative magnitude of a binary number.

Hence 2's complement of 96 represents -96.

$$So\ -96 = 1001\ 1111$$

$$118 + (-96) = 0111\ 0110$$

$$\begin{array}{r}
 + 1001\ 1111 \\
 \hline
 \underline{10001\ 0101}
 \end{array}$$

As a carry bit is present, 1 will be added to the result and presents of carry bit represents that the result is positive.

$$0001\ 0101 + 1 = 0001\ 0110$$

Decimal representation of 0001 0110 is 22. As carry bit is present the result is positive.

Example 2: Subtract 118 from 96.

$$118-96 = 96 + (-118)$$

Binary representation of 96= 0110 0000

Binary representation of 118= 0111 0110

Here 2's complement represents the negative magnitude of a binary number.

Hence 2's complement of 118 represents -118.

So $-118 = 1000\ 1010$

$96 + (-118) = 0110\ 0000$

$$\begin{array}{r} + 1000\ 1010 \\ \hline 1110\ 1010 \end{array}$$

As a carry bit is absent, 2's complement of this value will be the final result and absence of carry bit represents that the result is negative.

2's complement of $1110\ 1010 = 0001\ 0110$

Decimal representation of $0001\ 0110$ is 34. As carry bit is absent the result is negative i.e -34.

Binary Codes

Binary codes are codes which are represented in binary system with modification from the original ones. Below we will be seeing the following:

- Weighted Binary Systems
- Non Weighted Codes

Weighted Binary Systems

Weighted binary codes are those which obey the positional weighting principles, each position of the number represents a specific weight. The binary counting sequence is an example.

8421 Code/BCD Code

The BCD (Binary Coded Decimal) is a straight assignment of the binary equivalent. It is possible to assign weights to the binary bits according to their positions. The weights in the

BCD code are 8,4,2,1.

Example: The bit assignment 1001, can be seen by its weights to represent the decimal 9 because:

$$1 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1 = 9$$

2421 Code

This is a weighted code, its weights are 2, 4, 2 and 1. A decimal number is represented in 4-bit form and the total four bits weight is $2 + 4 + 2 + 1 = 9$. Hence the 2421 code represents the decimal numbers from 0 to 9.

5211 Code

This is a weighted code, its weights are 5, 2, 1 and 1. A decimal number is represented in 4-bit form and the total four bits weight is $5 + 2 + 1 + 1 = 9$. Hence the 5211 code represents the decimal numbers from 0 to 9.

Reflective Code

A code is said to be reflective when code for 9 is complement for the code for 0, and so is for 8 and 1 codes, 7 and 2, 6 and 3, 5 and 4. Codes 2421, 5211, and excess-3 are reflective, whereas the 8421 code is not.

Excess-3 Code

Excess-3 is a non-weighted code used to express decimal numbers. The code derives its name from the fact that each binary code is the corresponding 8421 code plus 0011(3).

Example: representation of 7 in Excess-3 code is :-

BCD code of 7 = 0111

Excess-3 code of 7 = BCD + 0011 = 1010

Gray Code

The gray code belongs to a class of codes called minimum change codes, in which only one bit in the code changes when moving from one code to the next. The Gray code is non weighted code, as the position of bit does not contain any weight. The gray code is are reflective digital code which has the special property that any two subsequent numbers codes differ by only one bit. This is also called a unit-distance code. In digital Gray code has got a special place.

Example: Write the Gray code of 7.

Step-1 BCD code of 7 = 0111

Step-2 Keep the MSB of BCD same and then add it with the next digit. Ignore the carry bit in each case.

$$\begin{array}{r} (0111)_2 = (0100)_G \\ \begin{array}{cccc} + & + & + & \\ \text{---} & \text{---} & \text{---} & \\ 0 & 1 & 1 & 1 \end{array} = \underline{0}100 \end{array}$$

Error Detecting and Correction Codes

For reliable transmission and storage of digital data, error detection and correction is required. Below are a few examples of codes which permit error detection and error correction after detection.

Error Detecting Codes

When data is transmitted from one point to another, like in wireless transmission, or it is just stored, like in hard disks and memories, there are chances that data may get corrupted. To detect these data errors, we use special codes, which are error detection codes.

Parity

In parity codes, every data byte, or nibble (according to how user wants to use it) is checked if they have even number of ones or even number of zeros. Based on this information an additional bit is appended to the original data. Thus if we consider 8-bit data, adding the parity bit will make it 9 bit long.

At the receiver side, once again parity is calculated and matched with the received parity (bit 9), and if they match, data is ok, otherwise data is corrupt.

There are two types of parity:

- Even parity: Checks if there is an even number of ones; if so, parity bit is zero. When the number of ones is odd then parity bit is set to 1.
- Odd Parity: Checks if there is an odd number of ones; if so, parity bit is zero. When number of ones is even then parity bit is set to 1.

Error-Correcting Codes

Error correcting codes not only detect errors, but also correct them. This is used normally in Satellite communication, where turn-around delay is very high as is the probability of data getting corrupt.

ECC (Error correcting codes) are used also in memories, networking, Hard disk, CDROM,

DVD etc. Normally in networking chips (ASIC), we have 2 Error detection bits and 1 Error correction bit.

Hamming Code

Hamming code adds a minimum number of bits to the data transmitted in a noisy channel, to be able to correct every possible one-bit error. It can detect (not correct) two bits errors and cannot distinguish between 1-bit and 2-bits inconsistencies. It can't – in general – detect 3(or more)-bits errors The idea is that the failed bit position in an n-bit string (which we'll call X) can be represented in binary with $\log_2(n)$ bits, hence we'll try to get it adding just $\log_2(n)$ bits.

ASCII Code

ASCII stands for American Standard Code for Information Interchange. It has become a world standard alphanumeric code for microcomputers and computers. It is a 7-bit code representing $2^7 = 128$ different characters. These characters represent 26 upper case letters

(A to Z), 26 lowercase letters (a to z), 10 numbers (0 to 9), 33 special characters and symbols and 33 control characters.

Boolean algebra and Logic Gates

The English mathematician George Boole (1815-1864) sought to give symbolic form to Aristotle's system of logic. Boole wrote a treatise on the subject in 1854, titled an Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities, which codified several rules of relationship between Mathematical quantities limited to one of two possible values: true or false, 1 or 0. His Mathematical system became known as Boolean algebra. All arithmetic operations performed with Boolean quantities have but one of two possible Outcomes: either 1 or 0. There is no such thing as $\| 2 \|$ or $\| -1 \|$ or $\| 1/2 \|$ in the Boolean world.

It is a world in which all other possibilities are invalid by fiat. As one might guess, this is not the kind of math you want to use when balancing a check book or calculating current through a resistor.

However, Claude Shannon of MIT fame recognized how Boolean algebra could be applied to on-and-off circuits, where all signals are characterized as either || high || (1) or || low || (0). His 1938 thesis, titled A Symbolic Analysis of Relay and Switching Circuits, put Boole's theoretical work to use in a way Boole never could have imagined, giving us a Powerful mathematical tool for designing and analysing digital circuits.

Like || normal || algebra, Boolean algebra uses alphabetical letters to denote variables.

Unlike || normal || algebra, though, Boolean variables are always CAPITAL letters, never lowercase.

Boolean Arithmetic

Let us begin our exploration of Boolean algebra by adding numbers together:

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 1$$

The first three sums make perfect sense to anyone familiar with elementary addition.

The Last sum, though, is quite possibly responsible for more confusion than any other

Single statement in digital electronics, because it seems to run contrary to the basic principles of mathematics. Well, it does contradict principles of addition for real numbers, but not for Boolean numbers. Remember that in the world of Boolean algebra, there are only two possible values for any quantity and for any arithmetic operation: 1 or 0. There is no such thing as || 2 || within the scope of Boolean values. Since the sum || 1 + 1 || certainly isn't 0, it must be 1 by process of elimination.

Principle of Duality

It states that every algebraic expression is deducible from the postulates of Boolean algebra and it remains valid if the operators & identity elements are interchanged. If the inputs of a NOR gate are inverted, we get a AND equivalent circuit. Similarly, when the inputs of a NAND gate are inverted, we get an OR equivalent circuit. This property is called duality.

Theorems of Boolean algebra and Identities

The theorems of Boolean algebra can be used to simplify many a complex Boolean expression and also to transform the given expression into a more useful and meaningful equivalent expression. The theorems are presented as pairs, with the two theorems in a given pair being the dual of each other. These theorems can be very easily verified by the method of 'perfect induction'. According to this method, the validity of the expression is tested for all possible combinations of values of

the variables involved. Also, since the validity of the theorem is based on its being true for all possible combinations of values of variables, there is no reason why a variable cannot be replaced with its complement, or vice versa, without disturbing the validity. Another important point is that, if a given expression is valid, its dual will also be valid.

Theorem 1 (Operations with '0' and '1')

(a) $0.X = 0$ and (b) $1+X = 1$

Where X is not necessarily a single variable – it could be a term or even a large expression.

Theorem 1(a) can be proved by substituting all possible values of X, that is, 0 and 1, into the given expression and checking whether the LHS equals the RHS:

· For $X = 0$, $LHS = 0.X = 0.0 = 0 = RHS$.

· For $X = 1$, $LHS = 0.1 = 0 = RHS$.

Thus, $0.X = 0$ irrespective of the value of X, and hence the proof.

Theorem 1(b) can be proved in a similar manner. In general, according to theorem 1, $0 \cdot (\text{Boolean expression}) = 0$ and $1 + (\text{Boolean expression}) = 1$.

1. For example: $0 \cdot (A.B + B.C + C.D) = 0$ and $1 + (A.B + B.C + C.D) = 1$, where A, B and C are Boolean variables.

Theorem 2 (Operations with '0' and '1')

(a) $1.X = X$ and (b) $0+X = X$

where X could be a variable, a term or even a large expression.

According to this theorem, ANDing a Boolean expression to '1' or ORing '0' to it makes no difference to the expression:

For $X = 0$, $LHS = 1.0 = 0 = RHS$.

For $X = 1$, $LHS = 1.1 = 1 = RHS$.

Also,

1. $(\text{Boolean expression}) = \text{Boolean expression}$ and $0 + (\text{Boolean expression}) = \text{Boolean expression}$.

For example,

$1.(A+B.C + C.D) = 0+(A+B.C +C.D) = A+B.C +C.D$

Theorem 3 (Idempotent or Identity Laws)

(a) $X.X.X \cdots X = X$ and (b) $X+X+X + \cdots +X = X$

Theorems 3(a) and (b) are known by the name of idempotent laws, also known as identity laws.

Theorem 3(a) is a direct outcome of an AND gate operation, whereas theorem 3(b) represents an OR gate operation when all the inputs of the gate have been tied together. The scope of idempotent laws can be expanded further by considering X to be a term or an expression. For example, let us apply idempotent laws to simplify the following Boolean expression:

$$\begin{aligned}(A.\bar{B}.\bar{B} + C.C).(A.\bar{B}.\bar{B} + A.\bar{B} + C.C) &= (A.\bar{B} + C).(A.\bar{B} + A.\bar{B} + C) \\ &= (A.\bar{B} + C).(A.\bar{B} + C) = A.\bar{B} + C\end{aligned}$$

Theorem 4 (Complementation Law)

(a) $X.\bar{X} = 0$ and (b) $X + \bar{X} = 1$

According to this theorem, in general, any Boolean expression when ANDed to its complement yields a '0' and when ORed to its complement yields a '1', irrespective of the complexity of the expression:

- For $X = 0, \bar{X} = 1$. Therefore, $X.\bar{X} = 0.1 = 0$.
- For $X = 1, \bar{X} = 0$. Therefore, $X.\bar{X} = 1.0 = 0$.

Hence, theorem 4(a) is proved. Since theorem 4(b) is the dual of theorem 4(a), its proof is implied.

The example below further illustrates the application of complementation laws:

$$(A + B.C)(\overline{A + B.C}) = 0 \quad \text{and} \quad (A + B.C) + \overline{A + B.C} = 1$$

Theorem 5 (Commutative property)

Mathematical identity, called a || property || or a || law || describes how differing variables relate to each other in a system of numbers. One of these properties is known as the commutative property, and it applies equally to addition and multiplication.

In essence, the commutative property tells us we can reverse the order of variables that are either added together or multiplied together without changing the truth of the expression:

Commutative property of addition

$$A + B = B + A$$

Commutative property of multiplication

$$AB = BA$$

Theorem 6 (Associative Property)

The Associative Property, again applying equally well to addition and multiplication.

This property tells us we can associate groups of added or multiplied variables together with parentheses without altering the truth of the equations.

Associative property of addition

$$A + (B + C) = (A + B) + C$$

Associative property of multiplication

$$A (BC) = (AB) C$$

Theorem 7 (Distributive Property)

The Distributive Property, illustrating how to expand a Boolean expression formed by the product of a sum, and in reverse shows us how terms may be factored out of Boolean sums-of-products:

Distributive property

$$A (B + C) = AB + AC$$

Theorem 8 (Absorption Law or Redundancy Law)

(a) $X+X.Y = X$ and (b) $X.(X+Y) = X$

The proof of absorption law is straightforward:

$$X+X.Y = X. (1+Y) = X.1 = X$$

Theorem 8(b) is the dual of theorem 8(a) and hence stands proved.

The crux of this simplification theorem is that, if a smaller term appears in a larger term, then the larger term is redundant. The following examples further illustrate the underlying concept:

$$A + A.\bar{B} + A.\bar{B}.\bar{C} + A.\bar{B}.C + \bar{C}.B.A = A$$

and

$$(\bar{A} + B + \bar{C}).(\bar{A} + B).(C + B + \bar{A}) = \bar{A} + B$$

De-Morgan's First Theorem

It States that the complement of the sum of the variables is equal to the product of the complement of each variable. This theorem may be expressed by the following Boolean expression.

$$\overline{A + B} = \bar{A}.\bar{B}$$

De-Morgan's Second Theorem

It states that the complement of the product of variables is equal to the sum of Complements of each individual variables || . Boolean expression for this theorem is

$$\overline{A.B} = \bar{A} + \bar{B}$$

Boolean Function

Boolean functions are represented in various forms. The two popular forms are truth tables and Venn diagram. Truth tables represent functions in a tabular form, while Venn diagram provide a graphic representation. In addition, there are two algebraic representation known as the standard form or canonical form.

Example: Truth table for $Z = AB' + A'C + A'B'C$

There are three variables present in the equation; A, B, C. Hence, there will be $2^3 = 8$ combinations of values. These eight combinations are shown in the first three columns of the truth table. These combinations corresponds to binary numbers 000 through 111.



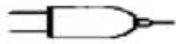
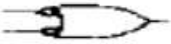
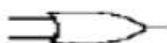








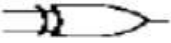


To evaluate Z in the example function, knowing the values for A,B,C at each row of the truth table, we should first generate the values for A' and B' and then generate the values of AB', A'C and A'B'C by ANDing the values in the appropriate columns for each row. Finally, we should derive the values of Z by ORing the values in the last three columns for each row. Note that evaluating A'B'C corresponds to ANDing A' and B' values, followed by ANDing the value of C.

$$Z = AB' + A'C + A'B'C$$

A	B	C	A'	B'	AB'	A'C	A'B'C	Z
0	0	0	1	1	0	0	0	0
0	0	1	1	1	0	1	1	1
0	1	0	1	0	0	0	0	0
0	1	1	1	0	0	1	0	1
1	0	0	0	1	1	0	0	1
1	0	1	0	1	1	0	0	1
1	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0

Complete Logic Sets

The basic logic gates are NOT, AND and OR. The complex logic functions like NAND, NOR, XOR, XNOR are created by using these basic logic gates. The last two are not standard terms; they stand for 'inverter' and 'buffer', respectively. The symbols for these gates and their corresponding Boolean expressions are given in Fig. 2.

Name	Expression	Symbol	Negative true symbol
AND	AB		
NAND	\overline{AB}		
OR	$A + B$		
NOR	$\overline{A + B}$		
INVERT	\overline{A}		
BUFFER	A		
XOR	$A \oplus B$		
XNOR	$\overline{A \oplus B}$		

All of the logical gate functions, as well as the Boolean relations discussed in the next section, follow from the truth tables for the AND and OR gates. A complete set of logic operations is one that allows us to create every possible logic function using only those in the set.

Basic Logic Operations

Boolean algebra is based on a set of logic operations that define basic functions. They are performed using one or more input variables and they result in a single output bit.

NOT Operation

Because they are allowed to possess only one of two possible values, either 1 or 0, each and every variable has a complement: the opposite of its value. For example, if variable A has a value of 0, then the complement of A has a value of 1. Boolean notation uses a bar above the variable character to denote complementation, like this:

$$\text{If: } A=0, \text{ then: } \overline{A}=1$$

$$\text{If: } A=1, \text{ then: } \overline{A}=0$$

In written form, the complement of A denoted as A -not or A -bar. Sometimes a

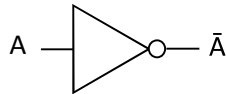
prime symbol is used to represent complementation. For example, A' would be the complement of A , much the same as using a prime symbol to denote differentiation in calculus

rather than the fractional notation dot. Usually, though, the || bar || symbol finds more wide spread use than the || prime || symbol, for reasons that will become more apparent later in this chapter.

Truth Table:

A	\bar{A}
0	1
1	0

Symbol:



OR Operation

Let us consider there are two input bits, A and B. each of two bits can assume a value of 0 or 1. So $2^2=4$ possible combinations can occur. This can be listed as AB= 00, 01, 10, 11.

A OR B can be restated as $A + B$. We usually employ the symbol “+” to denote OR operation.

$$A \text{ OR } B = A + B$$

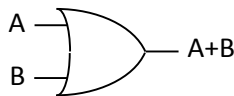
Example: If $A=1$, $B=1$ then,

$$A \text{ OR } B = A + B = 1$$

Truth Table:

A	B	$A + B$
0	0	0
0	1	1
1	0	1
1	1	1

Symbol



AND Operation

This operation detects the situation where all of the inputs are equal to 1 for this case. For two inputs A and B, we define the AND logic by writing

$$A \text{ AND } B = A.B$$

If both $A=1$ and $B=1$,
then

$$A \text{ AND } B = 1$$

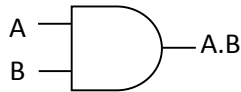
else

$$A \text{ AND } B = 0$$

Truth Table:

A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

Symbol



Universal Logic Gates

NAND Logic Gate

The name NAND is the shorten form of NOT-AND and means the output is defined as the complement of the AND gate. With the inputs A and B, the NAND operation is denoted as $\overline{A.B}$. This is equivalent to the statement

If either input is 0,

then

$$\overline{A.B} = 1$$

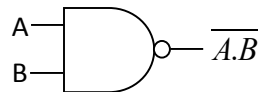
else

$$\overline{A.B} = 0$$

Truth Table:

A	B	$\overline{A.B}$
0	0	1
0	1	1
1	0	1
1	1	0

Symbol:



NOR Logic Gate

The name NOR is the shorten form of NOT-OR and means the output is defined as the complement of the OR gate. With the inputs A and B, the NOR operation is denoted as $\overline{A+B}$. This is equivalent to the statement

If either input is 0,

then

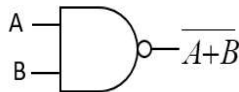
$$\overline{A+B} = 1$$

else
 $\overline{A+B}=0$

Truth Table:

A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

Symbol:



Algebraic Reduction

One common problem in combinational logic design is reduction of a logic expression to the “simplest” possible form; “simplest” usually means that we want to implement the function using the smallest no of gates. The reduction is accomplished by applying the basic identities in a step-by-step manner. Some basic rules are summarized in table given below to aid in the task.

OR Identities	AND Identities
$A+0=A$	$A.0=0$
$A+1=1$	$A.1=A$
$A+A=A$	$A.A=A$
$A+\bar{A}=1$	$A.\bar{A}=0$
$\overline{\overline{A}}=A$	-
$A+B=B+A$	$A.B=B.A$
$A+(B+C)=(A+B)+C$	$A.(B.C)=(A.B).C$
$A.(B+C)=A.B+A.C$	$A+(B.C)=(A+B).(A+C)$
$\overline{(A+B)}=\overline{A.B}$	$\overline{(A.B)}=\overline{A}+\overline{B}$
$A+A.B=A$	$A+\overline{A}.B=A+B$

Combinational Logic Design

Specifying the problem

Combinational logic deals with networks that use logic gates to combine the input variables as needed to produce logic functions. In combinational circuits the value of the input depends on the current values of the inputs. To design a combinational logic network, we usually start with a specified set of inputs to produce output.

Canonical form of Boolean Expression

An expanded form of Boolean expression, where each term contains all Boolean variables in their true or complemented form, is also known as the canonical form of the expression. As an illustration, $f(A,B,C) = \overline{A}.\overline{B}.\overline{C} + \overline{A}.\overline{B}.C + A.B.C$ is a Boolean function of three variables expressed in canonical form. This function after simplification reduces to $\overline{A}.\overline{B} + A.B.C$ and loses its canonical form.

Product-of-Sums Expressions

A product-of-sums expression contains the product of different terms, with each term being either a single literal or a sum of more than one literal. It can be obtained from the truth table by considering those input combinations that produce a logic '0' at the output. Each such input combination gives a term, and the product of all such terms gives the expression.

Different terms are obtained by taking the sum of the corresponding literals. Here '0' and '1' respectively mean the un-complemented and complemented variables, unlike sum-of products expressions where '0' and '1' respectively mean complemented and un-complemented variables.

Since each term in the case of the product-of-sums expression is going to be the sum of literals, this implies that it is going to be implemented using an OR operation. Now, an OR gate produces a logic '0' only when all its inputs are in the logic '0' state, which means that the first term corresponding to the second row of the truth table will be $A+B+C$. The product-of-sums Boolean expression for this truth table is given by transforming the given product-of-sums expression into an equivalent sum-of-products expression is a straight forward process. Multiplying out the given expression and carrying out the obvious simplification provides the equivalent sum-of-products expression:

$$F(x,y) = (\overline{x} + y).(x + \overline{y})$$

Sum of Products expression

A given sum-of-products expression can be transformed into an equivalent product-of sums expression by (a) taking the dual of the given expression, (b) multiplying out different terms to get the sum-of products form, (c) removing redundancy and (d) taking a dual to get the equivalent product-of-sums expression. As an illustration, let us find the equivalent product of sums expression of the sum-of products expression

$$A.B + \bar{A}.\bar{B}$$

The dual of the given expression = $(A + B).(\bar{A} + \bar{B})$:

$$(A + B).(\bar{A} + \bar{B}) = A.\bar{A} + A.\bar{B} + B.\bar{A} + B.\bar{B} = 0 + A.\bar{B} + B.\bar{A} + 0 = A.\bar{B} + \bar{A}.B$$

The dual of $(A.\bar{B} + \bar{A}.B) = (A + \bar{B}).(\bar{A} + B)$. Therefore

$$A.B + \bar{A}.\bar{B} = (A + \bar{B}).(\bar{A} + B)$$

Extracting Canonical Forms

Let us investigate this relationship by means of a specific case where we start with the function table shown below. The input variables are A, B, C and the output function is $f(A, B, C)$.

A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

From the table output f can be represented as

$$f = f_1 + f_2 + f_3 + f_4$$

where,

$$f_1 = \bar{A}\bar{B}C$$

$$f_2 = \bar{A}B\bar{C}$$

$$f_3 = A\bar{B}\bar{C}$$

$$f_4 = ABC$$

Hence the complete expression will be

$$f = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

Minterms and Maxterms

A minterm is the product of N distinct literals where each literal occurs exactly any Boolean expression may be expressed in terms of either minterms or maxterms. To do this we must first define the concept of a literal. A literal is a single variable within a term which may or may not be complemented. For an expression with N variables, minterms and maxterms are defined as follows

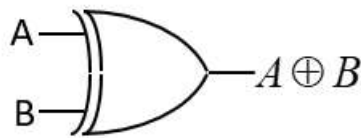
- once.
- A maxterm is the sum of N distinct literals where each literal occurs exactly once.

The Exclusive-OR and Equivalence Operation

By definition, the X-OR gate provides an output that is identical to that of the OR gate except for the case where both inputs are 1; the XOR output is 0 in this case. The name “exclusive-OR” arises from this exception in that the output is a 1 when only a single input is 1 exclusively, that is, by itself.

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Symbol:



The logical description of the gate can be extracted from the function table. The two cases that results in a 1 at the output are $AB=01$ and $AB=10$, so the function is given by

$$A \oplus B = A\bar{B} + \bar{A}B$$

The complement of the XOR function is the exclusive NOR (XNOR) operation; which can be denoted as $\overline{A \oplus B}$. The function is given by

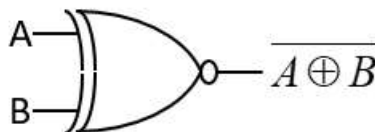
$$\overline{A \oplus B} = AB + \bar{A}\bar{B}$$

By reading off the SOP terms where the output is 1, this shows that it is only possible when both the inputs are same; i.e $A=B$. Because of this property, the XNOR is also referred to as the “equivalence function”.

Function Table:

A	B	$\overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

Symbol:



Karnaugh Map

Maurice Karnaugh, a telecommunications engineer, developed the Karnaugh map at Bell Labs in 1953 while designing digital logic based telephone switching circuits. Karnaugh maps reduce logic functions more quickly and easily compared to Boolean algebra. By reduce we mean simplify, reducing the number of gates and inputs. We like to simplify logic to a lowest cost form to save costs by elimination of components. We define lowest cost as being the lowest number of gates with the lowest number of inputs per gate. A Karnaugh map is a graphical representation of the logic system. It can be drawn directly from either minterm (sum-of-products) or maxterm (product-of-sums) Boolean expressions. Drawing a Karnaugh map from the truth table involves an additional step of writing the minterm or maxterm expression depending upon whether it is desired to have a minimized sum-of products or a minimized product of-sums expression.

Construction of a Karnaugh Map

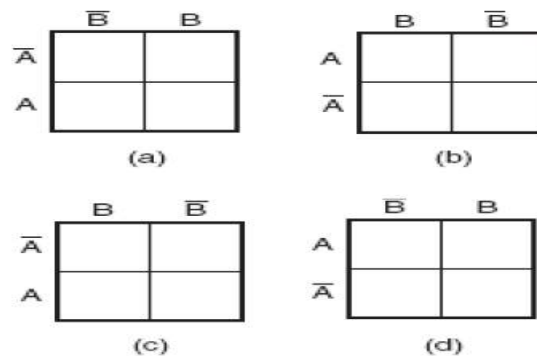
An n-variable Karnaugh map has 2^n squares, and each possible input is allotted a square. In the case of a minterm Karnaugh map, '1' is placed in all those squares for which the output is '1' and '0' is placed in all those squares for which the output is '0'. 0s are omitted for simplicity. An 'X' is placed in squares corresponding to 'don't care conditions. In the case of a maxterms Karnaugh map, a '1' is placed in all those squares for which the output is '0', and a '0' is placed for input entries corresponding to a '1' output. Again, 0s are omitted for simplicity, and an 'X' is placed in squares corresponding to 'don't care' conditions. The choice of terms identifying different rows and columns of a Karnaugh map is not unique for a given number of variables. The only condition to be satisfied is that the designation of adjacent rows and adjacent columns should be the same except for one of the literals being complemented. Also, the extreme rows and extreme columns are consider adjacent. Some of the possible designation styles for two, three and four variable minterm Karnaugh maps are shown in the figure below.

The style of row identification need not be the same as that of column identification as long as it meets the basic requirement with respect to adjacent terms. It is, however, accepted practice to adopt a uniform style of row and column identification. Also, the style shown in the figure below is more commonly used. A similar discussion applies for maxterms Karnaugh maps. Having drawn the Karnaugh map, the next step is to form groups of 1s as per the following guidelines:

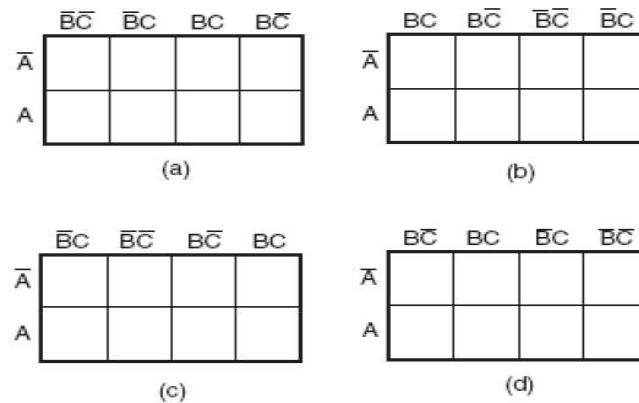
- Each square containing a '1' must be considered at least once, although it can be considered as often as desired.

- The objective should be to account for all the marked squares in the minimum number of groups.
- The number of squares in a group must always be a power of 2, i.e. groups can have 1, 2, 4, 8, 16, squares.
- Each group should be as large as possible, which means that a square should not be accounted for by itself if it can be accounted for by a group of two squares; a group of two squares should not be made if the involved squares can be included in a group of four squares and so on.
- ‘Don’t care’ entries can be used in accounting for all of 1-squares to make optimum groups. They are marked ‘X’ in the corresponding squares. It is, however, not necessary to account for all ‘don’t care’ entries. Only such entries that can be used to advantage should be used.

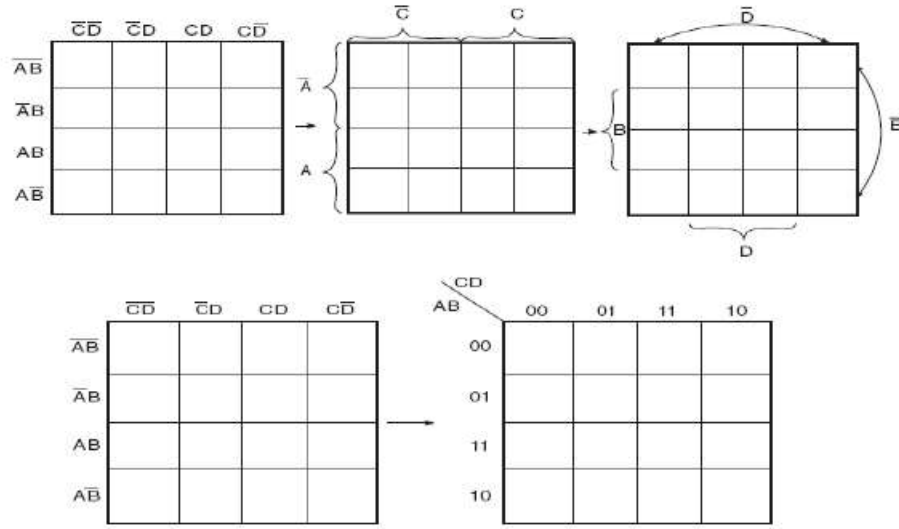
Two Variable K-Map



Three Variable K-Map



Four Variable K-Map



MODULE – II

Concept of logic components

Modern digital networks can be quite complicated, consisting of millions of logic gates. To design large system, we use the hierarchical approach where the network is broken down into smaller logic components that perform useful functions. The components themselves may be classified as basic components for system building even though they consist of smaller components or basic logic gates.

Binary Adder

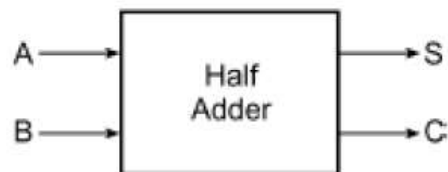
Half-Adder

A half-adder is an arithmetic circuit block that can be used to add two bits. Such a circuit thus has two inputs that represent the two bits to be added and two outputs, with one producing the SUM output and the other producing the CARRY. Figure shows the truth table of a half-adder, showing all possible input combinations and the corresponding outputs. The Boolean expressions for the SUM and CARRY outputs are given by the equations below

$$\text{SUM } S = A.\bar{B} + \bar{A}.B$$

$$\text{CARRY } C = A.B$$

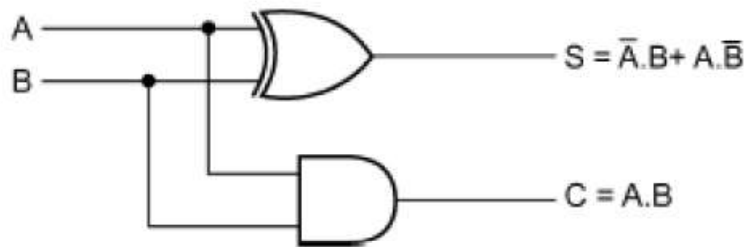
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



Truth Table of Half Adder

An examination of the two expressions tells that there is no scope for further simplification. While the first one representing the SUM output is that of an EX-OR gate, the second one representing the CARRY output is that of an AND gate. However, these two expressions can certainly be represented in different forms using various laws and theorems of Boolean algebra to illustrate the

flexibility that the designer has in hardware implementing as simple a combinational function as that of a half-adder.

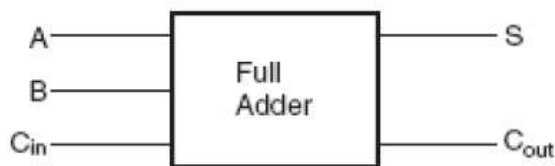


Logic Implementation of Half Adder

Although the simplest way to hardware-implement a half-adder would be to use a two input EX-OR gate for the SUM output and a two-input AND gate for the CARRY output, as shown in Fig. it could also be implemented by using an appropriate arrangement of either NAND or NOR gates.

Full Adder

A full adder circuit is an arithmetic circuit block that can be used to add three bits to produce a SUM and a CARRY output. Such a building block becomes a necessity when it comes to adding binary numbers with a large number of bits. The full adder circuit overcomes the limitation of the half-adder, which can be used to add two bits only. Let us recall the procedure for adding larger binary numbers. We begin with the addition of LSBs of the two numbers. We record the sum under the LSB column and take the carry, if any, forward to the next higher column bits. As a result, when we add the next adjacent higher column bits, we would be required to add three bits if there were a carry from the previous addition. We have a similar situation for the other higher column bits. Also until we reach the MSB. A full adder is therefore essential for the hardware implementation of an adder circuit capable of adding larger binary numbers. A half-adder can be used for addition of LSBs only.



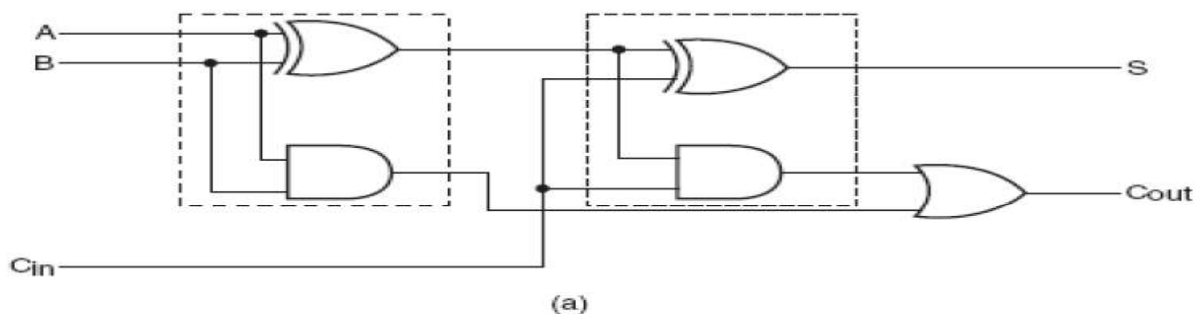
A	B	C _{in}	SUM (S)	C _{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Figure shows the truth table of a full adder circuit showing all possible input combinations and corresponding outputs. In order to arrive at the logic circuit for hardware implementation of a full adder, we will firstly write the Boolean expressions for the two output variables, that is, the SUM and CARRY outputs, in terms of input variables. These expressions are then simplified by using any of the simplification techniques described in the previous chapter. The Boolean expressions for the two output variables are given in Equation below for the SUM output (S) and in above Equation for the CARRY output (Cout):

$$S = \bar{A}.\bar{B}.C_{in} + \bar{A}.B.\bar{C}_{in} + A.\bar{B}.\bar{C}_{in} + A.B.C_{in}$$

$$C_{out} = \bar{A}.B.C_{in} + A.\bar{B}.C_{in} + A.B.\bar{C}_{in} + A.B.C_{in}$$

Boolean expression above can be implemented with a two-input EX-OR gate provided that one of the inputs is Cin and the other input is the output of another two-input EX-OR gate with A and B as its inputs. Similarly, Boolean expression above can be implemented by ORing two minterms. One of them is the AND output of A and B. The other is also the output of an AND gate whose inputs are Cin and the output of an EX-OR operation on A and B. The whole idea of writing the Boolean expressions in this modified form was to demonstrate the use of a half-adder circuit in building a full adder. Figure shows logic implementation of Equations above.



Subtractor and Multiplier

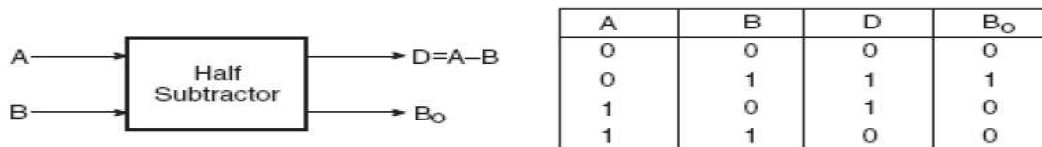
Half-Subtractor

We will study the use of adder circuits for subtraction operations in the following pages. Before we do that, we will briefly look at the counterparts of half-adder and full adder circuits in the half-subtractor and full subtractor for direct implementation of subtraction operations using logic gates.

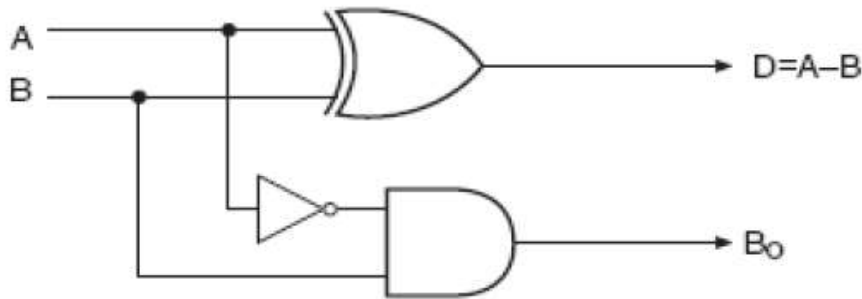
A half-subtractor is a combinational circuit that can be used to subtract one binary digit from another to produce a DIFFERENCE output and a BORROW output. The BORROW output here specifies whether a '1' has been borrowed to perform the subtraction. The truth table of a half-subtractor, as shown in Fig. explains this further. The Boolean expressions for the two outputs are given by the equations

$$D = \bar{A}.B + A.\bar{B}$$

$$B_o = \bar{A}.B$$



Half Subtractor



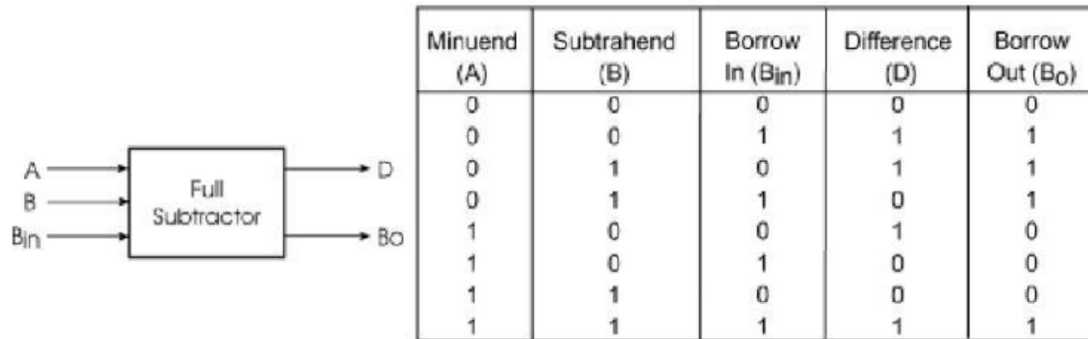
Logic Diagram of a Half Subtractor

Full-Subtractor

A full subtractor performs subtraction operation on two bits, a minuend and a subtrahend, and also takes into consideration whether a '1' has already been borrowed by the previous adjacent lower minuend bit or not. As a result, there are three bits to be handled at the input of a full subtractor, namely the two bits to be subtracted and a borrow bit designated as Bin .There are two outputs, namely the DIFFERENCE output D and the BORROW output Bo. The BORROW output bit tells whether the minuend bit needs to borrow a '1' from the next possible higher minuend bit. Figure shows the truth table of a full subtractor. The Boolean expressions for the two output variables are given by the equations

$$D = \bar{A}.\bar{B}.B_{in} + \bar{A}.B.\bar{B}_{in} + A.\bar{B}.\bar{B}_{in} + A.B.B_{in}$$

$$B_o = \bar{A}.\bar{B}.B_{in} + \bar{A}.B.\bar{B}_{in} + \bar{A}.B.B_{in} + A.B.B_{in}$$



Truth Table of Full Subtractor

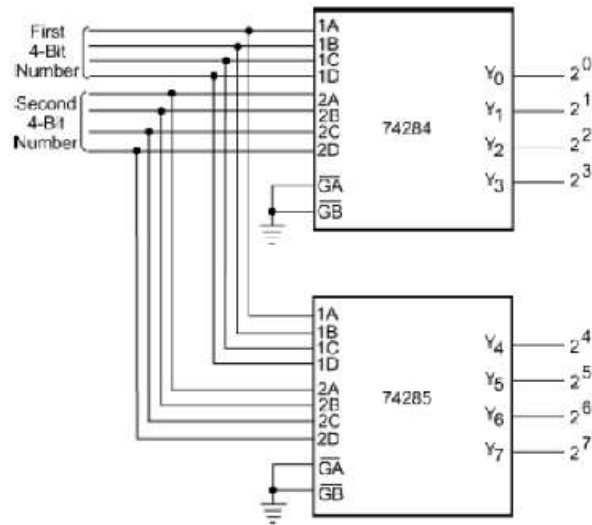
Binary Multiplier

Multiplication of binary numbers is usually implemented in microprocessors and microcomputers by using repeated addition and shift operations. Since the binary adders are designed to add only two binary numbers at a time, instead of adding all the partial products at the end, they are added two at a time and their sum is accumulated in a register called the accumulator register. Also, when the multiplier bit is '0', that very partial product is ignored, as an all '0' line does not affect the final result. The basic hardware arrangement of such a binary multiplier would comprise shift registers for the multiplicand and multiplier bits, an accumulator register for storing partial products, a binary parallel adder and a clock pulse generator to time various operations.

Binary multipliers are also available in IC form. Some of the popular type numbers in the TTL family include 74261 which is a 2 × 4 bit multiplier (a four-bit multiplicand designated

as B0, B1, B2, B3 and B4, and a two-bit multiplier designated as M0, M1 and M2. The MSBs

B4 and M2 are used to represent signs. 74284 and 74285 are 4 × 4 bit multipliers. They can be used together to perform high-speed multiplication of two four-bit numbers. Figure shows the arrangement. The result of multiplication is often required to be stored in a register. The size of this register (accumulator) depends upon the number of bits in the result, which at the most can be equal to the sum of the number of bits in the multiplier and multiplicand. Some multipliers ICs have an in-built register.



4 x 4 Multiplier

Magnitude comparator

A magnitude comparator is a combinational circuit that compares two given numbers and determines whether one is equal to, less than or greater than the other. The output is in the form of three binary variables representing the conditions $A = B$, $A > B$ and $A < B$, if A and B are the two numbers being compared. Depending upon the relative magnitude of the two numbers, the relevant output changes state. If the two numbers, let us say, are four-bit binary numbers and are designated as $(A_3 A_2 A_1 A_0)$ and $(B_3 B_2 B_1 B_0)$, the two numbers will be equal if all pairs of significant digits are equal, that is, $A_3 = B_3$, $A_2 = B_2$, $A_1 = B_1$ and $A_0 = B_0$. In order to determine whether A is greater than or less than B we inspect the relative magnitude of pairs of significant digits, starting from the most significant position. The comparison is done by successively comparing the next adjacent lower pair of digits if the digits of the pair under examination are equal. The comparison continues until a pair of unequal digits is reached. In the pair of unequal digits, if $A_i = 1$ and $B_i = 0$, then $A > B$, and if $A_i = 0$, $B_i = 1$ then $A < B$. If X , Y and Z are three variables respectively representing the $A = B$, $A > B$ and $A < B$ conditions, then the Boolean expression representing these conditions are given by the equations

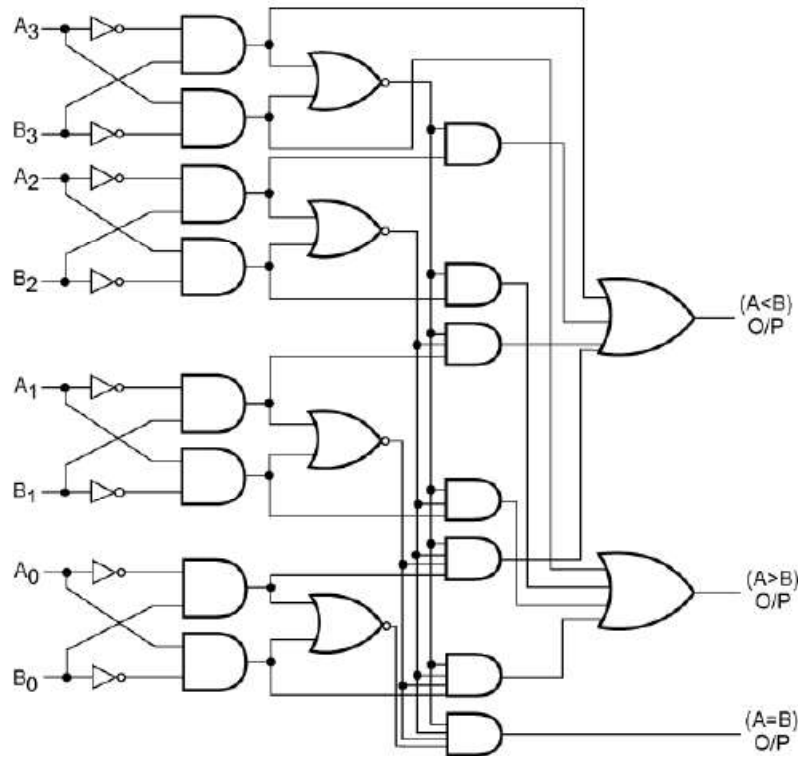
$$X = x_3 \cdot x_2 \cdot x_1 \cdot x_0 \quad \text{where } x_i = A_i \cdot B_i + \overline{A_i} \cdot \overline{B_i}$$

$$Y = A_3 \cdot \overline{B_3} + x_3 \cdot A_2 \cdot \overline{B_2} + x_3 \cdot x_2 \cdot A_1 \cdot \overline{B_1} + x_3 \cdot x_2 \cdot x_1 \cdot A_0 \cdot \overline{B_0}$$

$$Z = \overline{A_3} \cdot B_3 + x_3 \cdot \overline{A_2} \cdot B_2 + x_3 \cdot x_2 \cdot \overline{A_1} \cdot B_1 + x_3 \cdot x_2 \cdot x_1 \cdot \overline{A_0} \cdot B_0$$

Let us examine equations. x_3 will be '1' only when both A_3 and B_3 are equal. Similarly, conditions for x_2 , x_1 and x_0 to be '1' respectively are equal A_2 and B_2 , equal A_1 and B_1 and equal A_0 and B_0 . ANDing of x_3 , x_2 , x_1 and x_0 ensures that X will be '1' when x_3 , x_2 , x_1 and x_0 are in the logic '1' state. Thus, $X = 1$ means that $A = B$. On similar lines, it can be visualized that equations and respectively represent $A > B$ and $A < B$ conditions. Figure shows the logic diagram of a four-bit magnitude comparator.

Magnitude comparators are available in IC form. For example, 7485 is a four bit magnitude comparator of the TTL logic family. IC 4585 is a similar device in the CMOS family. 7485 and 4585 have the same pin connection diagram and functional table. The logic circuit inside these devices determines whether one four-bit number, binary or BCD, is less than, equal to or greater than a second four-bit number. It can perform comparison of straight binary and straight BCD (8-4-2-1) codes. These devices can be cascaded together to perform operations on larger bit numbers without the help of any external gates. This is facilitated by three additional inputs called cascading or expansion inputs available on the IC. These cascading inputs are also designated as $A = B$, $A > B$ and $A < B$ inputs. Cascading of individual magnitude comparators of the type 7485 or 4585 is discussed in the following paragraphs. IC 74AS885 is another common magnitude comparator. The device is an eight bit magnitude comparator belonging to the advanced Schottky TTL family. It can perform high-speed arithmetic or logic comparisons on two eight-bit binary or 2's complement numbers and produces two fully decoded decisions at the output about one number being either greater than or less than the other. More than one of these devices can also be connected in a cascade arrangement to perform comparison of numbers of longer lengths.



Four Bit Magnitude Comparator

Decoders and Encoders

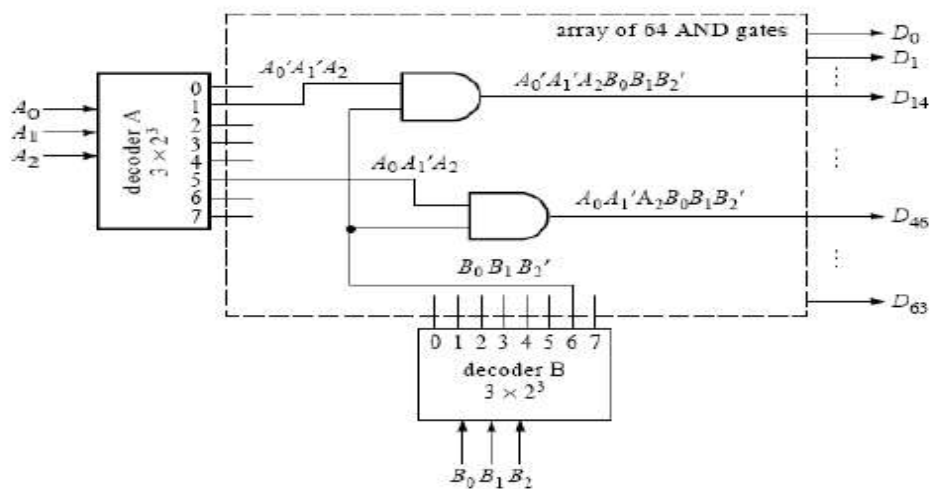
The previous section began by discussing an application: Given $2n$ data signals, the problem is to select, under the control of n select inputs, sequences of these $2n$ data signals to send out serially on a communications link. The reverse operation on the receiving end of the communications link is to receive data serially on a single line and to convey it to one of $2n$ output lines. This again is controlled by a set of control inputs. It is this application that needs only one input line; other applications may require more than one. We will now investigate such a generalized circuit.

Conceivably, there might be a combinational circuit that accepts n inputs (not necessarily 1, but a small number) and causes data to be routed to one of many, say up to $2n$, outputs. Such circuits have the generic name decoder. Semantically, at least, if something is to be decoded, it must have previously been encoded, the reverse operation from decoding. Like a multiplexer, an encoding circuit must accept data from a large number of input lines and convert it to data on a smaller number of output lines (not necessarily just one). This section will discuss a number of implementations of decoders and encoders.

n-to-2n-Line Decoder

In the demultiplexer circuit in Figure, suppose the data input line is removed. (Draw the circuit for yourself.) Each AND gate now has only n (in this case three) inputs, and there are $2n$ (in this case eight) outputs. Since there isn't a data input line to control, what used to be control inputs no longer serve that function. Instead, they are the data inputs to be decoded.

This circuit is an example of what is called an n -to- $2n$ -line decoder. Each output represents a minterm. Output k is 1 whenever the combination of the input variable values is the binary equivalent of decimal k . Now suppose that the data input line from the demultiplexer in Figure 16 is not removed but retained and viewed as an enable input. The decoder now operates only when the enable x is 1. Viewed conversely, an n -to- $2n$ -line decoder with an enable input can also be used as a demultiplexer, where the enable becomes the serial data input and the data inputs of the decoder become the control inputs of the demultiplexer.⁷ Decoders of the type just described are available as integrated circuits (MSI); $n = 3$ and $n = 4$ are quite common. There is no theoretical reason why n can't be increased to higher values. Since, however, there will always be practical limitations on the fan-in (the number of inputs that a physical gate can support), decoders of higher order are often designed using lower-order decoders interconnected with a network of other gates.



Encoder

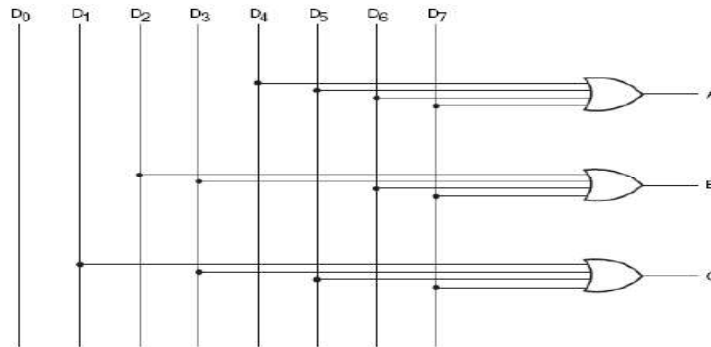
An encoder is a combinational circuit that performs the inverse operation of a decoder. If a device output code has fewer bits than the input code has, the device is usually called an encoder. e.g. $2n$ -

to-n, priority encoders. The simplest encoder is a 2^n -to-n binary encoder, where it has only one of 2^n inputs =1 and the output is the n-bit binary number corresponding to the active input.

Priority Encoder

A priority encoder is a practical form of an encoder. The encoders available in IC form are all priority encoders. In this type of encoder, a priority is assigned to each input so that, when more than one input is simultaneously active, the input with the highest priority is encoded. We will illustrate the concept of priority encoding with the help of an example.

Let us assume that the octal to-binary encoder described in the previous paragraph has an input priority for higher-order digits. Let us also assume that input lines D2, D4 and D7 are all simultaneously in logic '1' state. In that case, only D7 will be encoded and the output will be 111.



The truth table of such a priority encoder will then be modified to what is shown above in truth table. Looking at the last row of the table, it implies that, if $D_7 = 1$, then, irrespective of the logic status of other inputs, the output is 111 as D₇ will only be encoded. As another example, Fig. shows the logic symbol and truth table of a 10-line decimal to four-line BCD encoder providing priority encoding for higher-order digits, with digit 9 having the highest priority. In the functional table shown, the input line with highest priority having a LOW on it is encoded irrespective of the logic status of the other input lines.

Octal to binary encoder

D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	A	B	C
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

Truth table of encoder

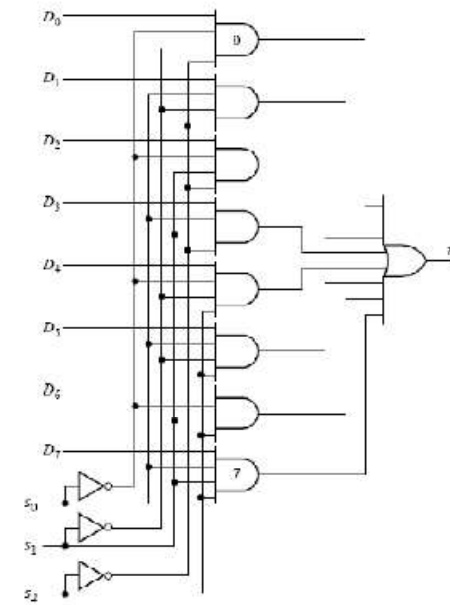
MULTIPLEXERS

Data generated in one location is to be used in another location; a method is needed to transmit it from one location to another through some communications channel. The data is available, in parallel, on many different lines but must be transmitted over a single communications link.

A mechanism is needed to select which of the many data lines to activate sequentially at any one time so that the data this line carries can be transmitted at that time. This process is called multiplexing. An example is the multiplexing of conversations on the telephone system. A number of telephone conversations are alternately switched onto the telephone line many times per second. Because of the nature of the human auditory system, listeners cannot detect that what they are hearing is chopped up and that other people's conversations are interspersed with their own in the transmission process. Needed at the other end of the communications link is a device that will undo the multiplexing: a demultiplexer. Such a device must accept the incoming serial data and direct it in parallel to one of many output lines. The interspersed snatches of telephone conversations, for example, must be sent to the correct listeners.

A digital multiplexer is a circuit with $2n$ data input lines and one output line. It must also have a way of determining the specific data input line to be selected at any one time.

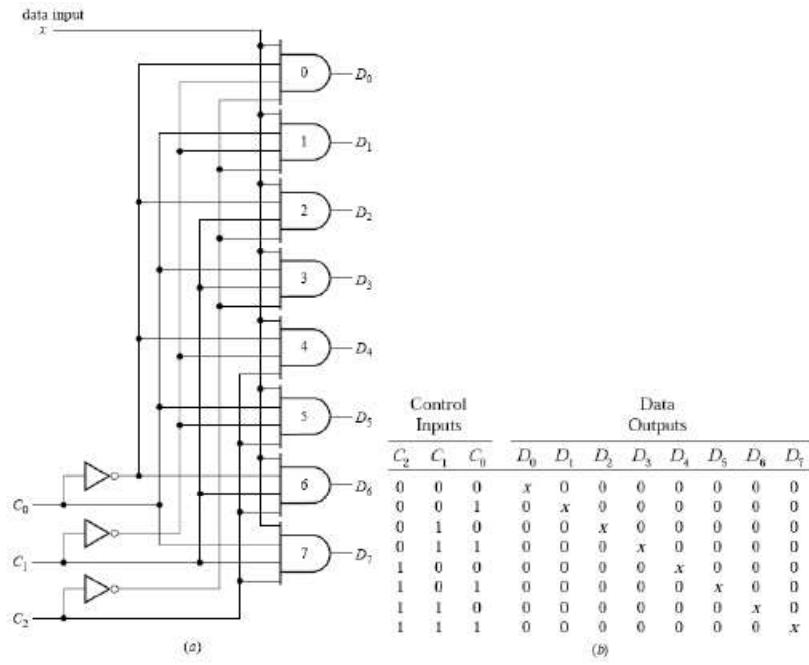
This is done with n other input lines, called the select or selector inputs, whose function is to select one of the $2n$ data inputs for connection to the output. A circuit for $n = 3$ is shown in Figure below. The n selector lines have $2^n = 8$ combinations of values that constitute binary select numbers



Multiplexer with eight data inputs

Demultiplexers

The demultiplexer shown there is a single-input, multiple-output circuit. However, in addition to the data input, there must be other inputs to control the transmission of the data to the appropriate data output line at any given time. Such a demultiplexer circuit having eight output lines is shown in Figure 16a. It is instructive to compare this demultiplexer circuit with the multiplexer circuit in Figure 13. For the same number of control (select) inputs, there are the same number of AND gates. But now each AND gate output is a circuit output. Rather than each gate having its own separate data input, the single data line now forms one of the inputs to each AND gate, the other AND inputs being control inputs.



A demultiplexer circuit (a) and its truth table (b).

Synchronous Sequential logic

Latches

The following 3 figures are equivalent representations of a simple circuit. In general these are called flip-flops. Specially, these examples are called SR (set-reset) flip-flops, or SR latches.

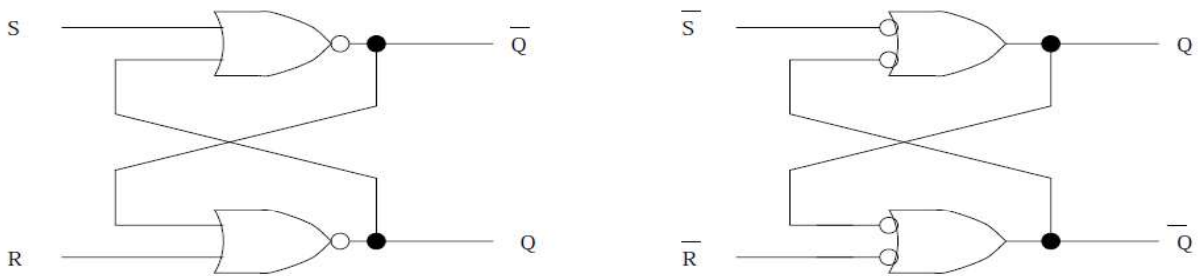


Figure 1: Two equivalent versions of an SR flip-flop (or “SR latch”).

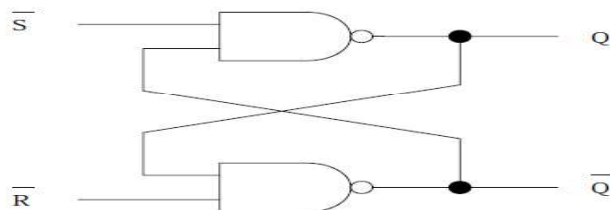


Figure 2: Yet another equivalent SR flip-flop

S	\bar{S}	R	\bar{R}	Q	\bar{Q}
1	0	0	1	1	0
0	1	1	0	0	1
0	1	0	1	Prev. value	Prev. value
1	0	1	0	0	0

The state described by the last row is clearly problematic, since Q and \bar{Q} should not be the same value. Thus the $S=R=1$ inputs should be avoided.

From the truth table, we can develop a sequence such as the following:

$R=0, S=1 \Rightarrow Q=1$ (Set)

$R=0, S=0 \Rightarrow Q=1$ ($Q=1$ state retained)

$R=1, S=0 \Rightarrow Q=0$ (Reset)

$R=0, S=0 \Rightarrow Q=0$ ($Q=0$ state retained)

In alternative language, the first operation “writes” a true state into one bit of memory. It can subsequently be “read” until it is erased by the reset operation of the third line.

Flip Flops

The flip-flop is an important element of such circuits. It has the interesting property of an SR Flip-flop has two inputs: S for setting and R for Resetting the flip- flop : It can be set to a state which is retained until explicitly reset.

R-S Flip-Flop

A flip-flop, as stated earlier, is a bistable circuit. Both of its output states are stable. The circuit remains in a particular output state indefinitely until something is done to change that output status. Referring to the bistable multivibrator circuit discussed earlier, these two states were those of the output transistor in saturation (representing a LOW output) and in cut-off (representing a HIGH output). If the LOW and HIGH outputs are respectively regarded as ‘0’ and ‘1’, then the output can either be a ‘0’ or a ‘1’. Since either a ‘0’ or a ‘1’ can be held indefinitely until the circuit is appropriately triggered to go to the other state, the circuit is said to have memory. It is capable of storing one binary digit or one bit of digital information. Also, if we recall the functioning of the bistable multivibrator circuit, we find that, when one of the transistors was in saturation, the other was in cut-off. This implies that, if we had taken outputs from the collectors of both transistors, then the two outputs would be complementary.

J-K Flip-Flop

A J-K flip-flop behaves in the same fashion as an R-S flip-flop except for one of the entries in the function table. In the case of an R-S flip-flop, the input combination $S = R = 1$ (in the case of a flip-flop with active HIGH inputs) and the input combination $S = R = 0$ (in the case of a flip-flop with active LOW inputs) are prohibited. In the case of a J-K flip-flop with active HIGH inputs, the output of the flip-flop toggles, that is, it goes to the other state, for $J = K = 1$. The output toggles for $J = K = 0$ in the case of the flip-flop having active LOW inputs. Thus, a J-K flip-flop overcomes the problem of a forbidden input combination of the R-S flip-flop. Figures below respectively show the circuit symbol of level-triggered J-K flip-flops with active HIGH and active LOW inputs, along with their function tables.

The characteristic tables for a J-K flip-flop with active HIGH J and K inputs and a J-K flip-flop with active LOW J and K inputs are respectively shown in Figs (a) and (b). The corresponding Karnaugh maps are shown in Fig below for the characteristics table of Fig and in below for the characteristic table below. The characteristic equations for the Karnaugh maps of below figure is shown next FIG

$$Q_{n+1} = J\overline{Q}_n + \overline{K}Q_n$$

$$Q_{n+1} = \overline{J}Q_n + K\overline{Q}_n$$

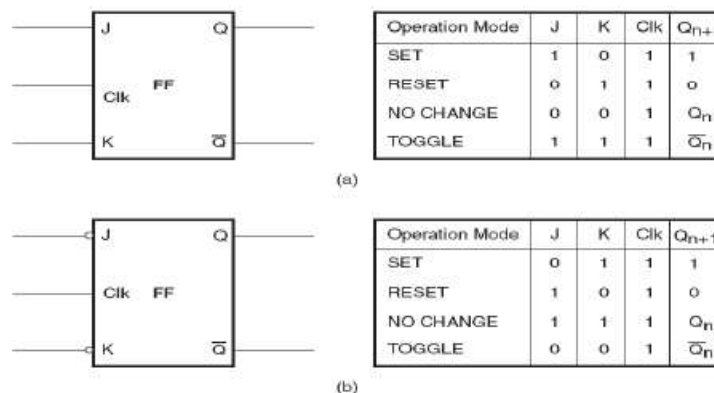


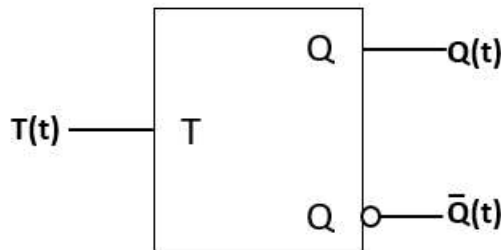
FIG a. JK flip flop with active high inputs, b. JK flip flop with active low inputs

Toggle Flip-Flop (T Flip-Flop)

The output of a toggle flip-flop, also called a T flip-flop, changes state every time it is triggered at its T input, called the toggle input. That is, the output becomes '1' if it was

'0' and '0' if it was '1'. Positive edge-triggered and negative edge-triggered T flip-flops, along with their function tables. If we consider the T input as active when HIGH, the characteristic table of such a flip-flop is shown in Fig. If the T input were active when LOW, then the characteristic table would be as shown in Fig. The Karnaugh maps for the characteristic tables of Figs shown respectively. The characteristic equations as written from the Karnaugh maps are as follows:

$$Q_{n+1} = T \cdot Q_n + \bar{T} \cdot \bar{Q}_n$$



T(t)	Q(t+T)
0	$Q(t)$
1	$\bar{Q}(t)$

D Flip-Flop

A D flip-flop, also called a delay flip-flop, can be used to provide temporary storage of one bit of information. Figure shows the circuit symbol and function table of a negative edge-triggered D flip-flop. When the clock is active, the data bit (0 or 1) present at the D input is transferred to the output. In the D flip-flop of Fig the data transfer from D input to Q output occurs on the negative-going (HIGH-to-LOW) transition of the clock input. The D input can acquire new status.



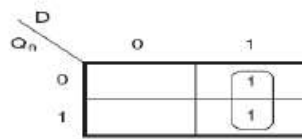
(a)

D	Clk	Q
0		0
1		1

(b)

Q_n	D	Q_{n+1}
0	0	0
0	1	1
1	0	0
1	1	1

(c)



$$Q_{n+1} = D$$

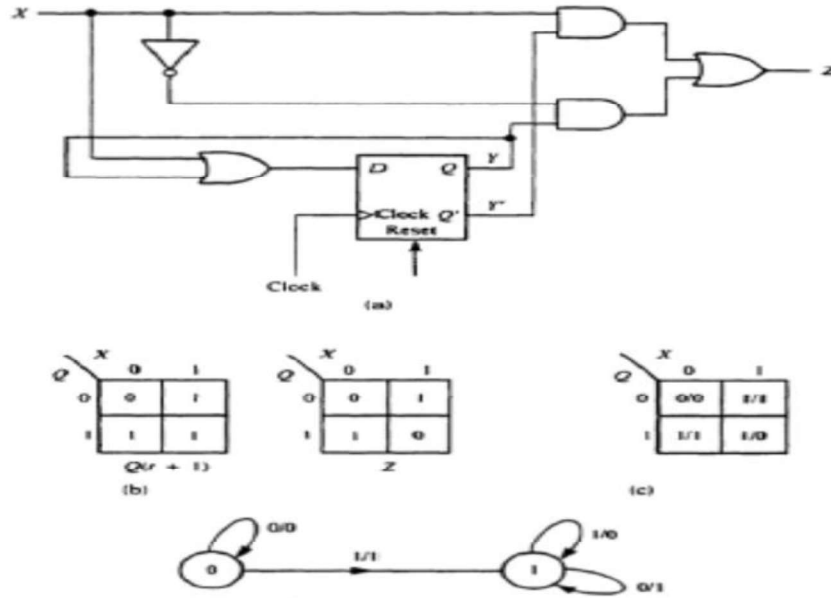
(d)

Analysis of Clocked Sequential circuits

The analysis of a synchronous sequential circuit is the process of determining the functional relation that exists between its output, its inputs and its internal state. The contents of all the flip-flops in the circuit combined determine the internal state of the circuit. Thus, the circuit contains n flip-flops, it can be in one of the 2^n states. Knowing the present state of the circuit and the input values at any time t , we should be able to derive its next state (i.e. the state at time $t+1$) and the output produced by the circuit at t .

A sequential circuit can be described completely by a state table that is very similar to the one shown for flip-flops. For a circuit with n flip-flops, there will be 2^n rows in the state table. If there are m inputs to the circuit then there will be 2^m no of columns. In the intersection of each row and column the next state and the output information will be recorded.

A state diagram is a graphical representation of state table, in which each state is represented as a circle and the state transitions are represented as arrows. Analysing a sequential circuit thus corresponds to generating the state table and state diagram for the circuit. The state table and state diagram can be used to determine the output sequence generated by the circuit for a given input sequence if the initial condition is known. Usually the power-up circuits are used to the appropriate state when the power is turned on.



(a) Sequential circuit analysis (b) Transition table (c) State diagram

Counters

In digital logic and computing, a counter is a device which stores (and sometimes displays) the number of times a particular event or process has occurred, often in relationship to a clock signal.

In practice, there are two types of counters:

- up counters which increase (increment) in value
- down counters which decrease (decrement) in value

Counters Types

In electronics, counters can be implemented quite easily using register-type circuits such as the flip-flop, and a wide variety of designs exist, e.g:

- Asynchronous (ripple) counters
- Synchronous counters
- Johnson counters
- Decade counters
- Up-Down counters
- Ring counters

Each is useful for different applications. Usually, counter circuits are digital in nature, and count in binary, or sometimes binary coded decimal. Many types of counter circuit are available as digital building blocks, for example a number of chips in the 4000 series implement different counters.

Asynchronous (ripple) counters

The simplest counter circuit is a single D-type flip flop, with its D (data) input fed from its own inverted output. This circuit can store one bit, and hence can count from zero to one before it overflows (starts over from 0). This counter will increment once for every clock cycle and takes two clock cycles to overflow, so every cycle it will alternate between a transition from 0 to 1 and a transition from 1 to 0. Notice that this creates a new clock with a 50% duty cycle at exactly half the frequency of the input clock. If this output is then used as the clock signal for a similarly arranged D flip flop (remembering to invert the output to the input), you will get another 1 bit counter that counts half as fast. Putting them together yields a two bit counter:

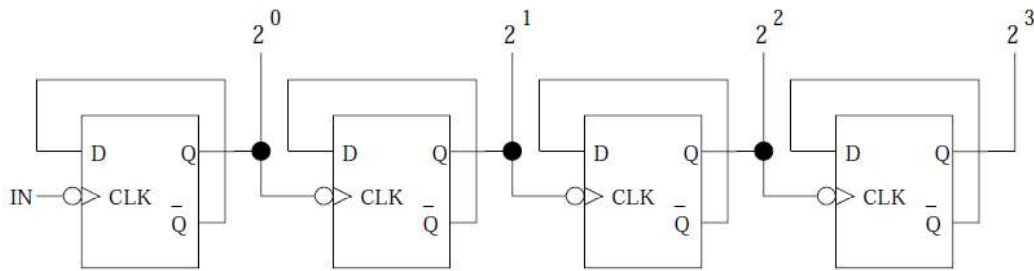


Figure : Asynchronous (“ripple”) counter made from cascaded D-type flip-flops.

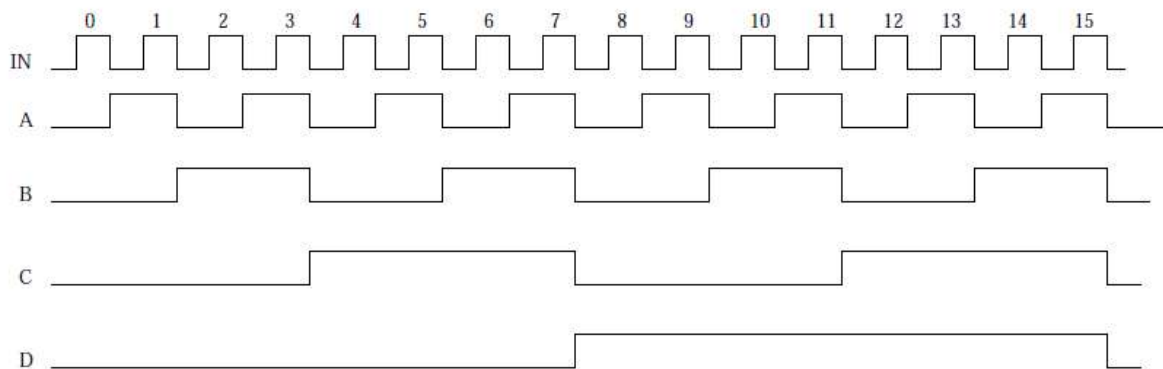


Figure : Waveforms generated by the ripple counter.

Decade counters

Decade counters are a kind of counter that counts in tens rather than having a binary representation. Each output will go high in turn, starting over after ten outputs have occurred. This type of circuit

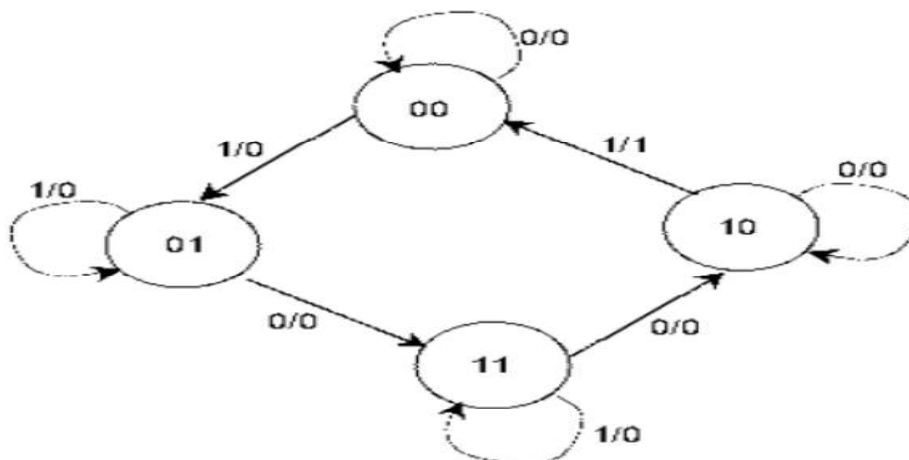
finds applications in multiplexers and demultiplexers, or wherever a scanning type of behaviour is useful. Similar counters with different numbers of outputs are also common.

Ring or Up-Down Counters

It is a combination of up counter and down counter, counting in straight binary sequence. There is an up-down selector. If this value is kept high, counter increments binary value and if the value is low, then counter starts decrementing the count. The Down counters are made by using the complemented output to act as the clock for the next flip-flop in the case of Asynchronous counters. An Up counter is constructed by linking the Q out of the J-K Flip flop and putting it into a Negative Edge Triggered Clock input. A Down Counter is constructed by taking the Q output and putting it into a Positive Edge Triggered input Ring Counters a ring counter is a counter that counts up and when it reaches the last number that is designed to count up to, it will reset itself back to the first number. For example, a ring counter that is designed using 3 JK Flip Flops will count starting from 001 to 010 to 100 and back to 001. It will repeat itself in a 'Ring' shape and thus the name Ring Counter is given.

State Diagram

In addition to graphical symbols, tables or equations, flip-flops can also be represented graphically by a state diagram. In this diagram, a state is represented by a circle, and the transition between states is indicated by directed lines (or arcs) connecting the circles. An example of a state diagram is shown in Figure 3 below.



The binary number inside each circle identifies the state the circle represents. The directed lines are labelled with two binary numbers separated by a slash (/). The input value that causes the state transition is labelled first. The number after the slash symbol / gives the value of the output. For

example, the directed line from state 00 to 01 is labelled 1/0, meaning that, if the sequential circuit is in a present state and the input is 1, then the next state is 01 and the output is 0. If it is in a present state 00 and the input is 0, it will remain in that state. A directed line connecting a circle with itself indicates that no change of state occurs. The state diagram provides exactly the same information as the state table and is obtained directly from the state table.

MODULE-III

Shift register

In digital circuits a shift register is a group of flip flops set up in a linear fashion which have their inputs and outputs connected together in such a way that the data is shifted down the line when the circuit is activated.

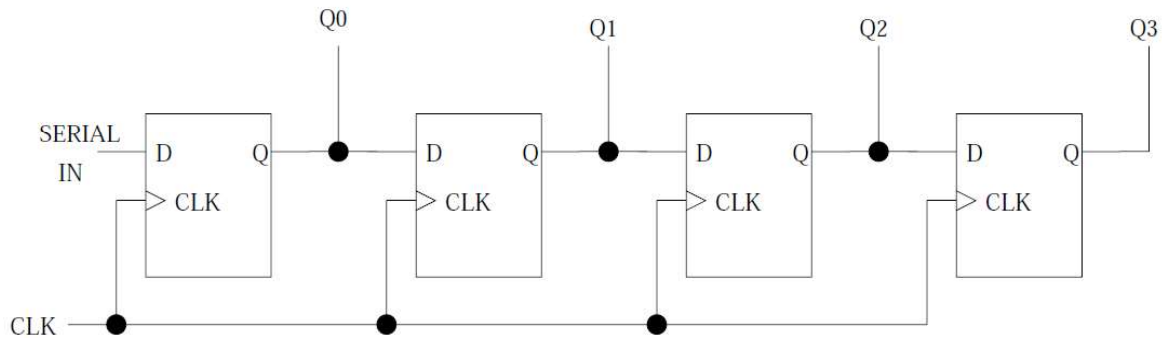


Figure : 4-bit shift register.

Memory & Programmable logic

The important common element of the memories we will study is that they are random access memories, or RAM. This means that each bit of information can be individually stored or retrieved with a valid input address. This is to be contrasted with sequential memories in which bits must be stored or retrieved in a particular sequence, for example with data storage on magnetic tape. Unfortunately the term RAM has come to have a more specific meaning: A memory for which bits can both be easily stored or retrieved ("written to" or "read from").

Classification of memories

Random Access Memory (RAM)

In general, refers to random access memory. All of the devices we are considering to be "memories" (RAM, ROM, etc.) are random access. The term RAM has also come to mean memory which can be both easily written to and read from.

RAM has three basic building blocks, namely an array of memory cells arranged in rows and columns with each memory cell capable of storing either a '0' or a '1', an address decoder and a

read/write control logic. Depending upon the nature of the memory cell used, there are two types of RAM, namely static RAM (SRAM) and dynamic RAM (DRAM). In SRAM, the memory cell is essentially a latch and can store data indefinitely as long as the DC power is supplied. DRAM on the other hand, has a memory cell that stores data in the form of charge on a capacitor. Therefore, DRAM cannot retain data for long and hence needs to be refreshed periodically. SRAM has a higher speed of operation than DRAM but has a smaller storage capacity.

Static RAM

These essentially are arrays of flip-flops. They can be fabricated in ICs as large arrays of (tint flip-flops.) "SRAM" is intrinsically somewhat faster than dynamic RAM.

Dynamic RAM.

Uses capacitor arrays. Charge put on a capacitor will produce a HIGH bit if its voltage $V = Q/C$ exceeds the threshold for the logic standard in use. Since the charge will "leak" through the resistance of the connections in times of order 1 msec, the stored information must be continuously refreshed (hence the term "dynamic"). Dynamic RAM can be fabricated with more bits per unit area in an IC than static RAM. Hence, it is usually the technology of choice for most large-scale IC memories.

Read-only memory.

Information cannot be easily stored. The idea is that bits are initially stored and are never changed thereafter. As an example, it is generally prudent for the instructions used to initialize a computer upon initial power-up to be stored in ROM. The following terms refer to versions of ROM for which the stored bits can be over-written, but not easily.

Programmable ROM.

Bits can be set on a programming bench by burning fusible links, or equivalent. This technology is also used for programmable array logic (PALs), which we will briefly discuss in class.

ROM Organization

A circuit for implementing one or more switching functions of several variables was described in the preceding section and illustrated in Figure 20. The components of the circuit are

- An $n \times 2^n$ decoder, with n input lines and 2^n output lines
- One or more OR gates, whose outputs are the circuit outputs

- An interconnection network between decoder outputs and OR gate inputs

The decoder is an MSI circuit, consisting of 2^n n -input AND gates, that produces all the minterms of n variables. It achieves some economy of implementation, because the same decoder can be used for any application involving the same number of variables. What is special to any application is the number of OR gates and the specific outputs of the decoder that become inputs to those OR gates. Whatever else can be done to result in a general-purpose circuit would be most welcome. The most general-purpose approach is to include the maximum number of OR gates, with provision to interconnect all 2^n outputs of the decoder with the inputs to every one of the OR gates. Then, for any given application, two things would have to be done:

- The number of OR gates used would be fewer than the maximum number, the others remaining unused.
- Not every decoder output would be connected to all OR gate inputs. This scheme would be terribly wasteful and doesn't sound like a good idea. Instead, suppose a smaller number, m , is selected for the number of OR gates to be included, and an interconnection network is set up to interconnect the 2^n decoder outputs to the m OR gate inputs. Such a structure is illustrated in above figure. It is an LSI combinational circuit with n inputs and m outputs that, for reasons that will become clear shortly, is called a read-only memory (ROM).

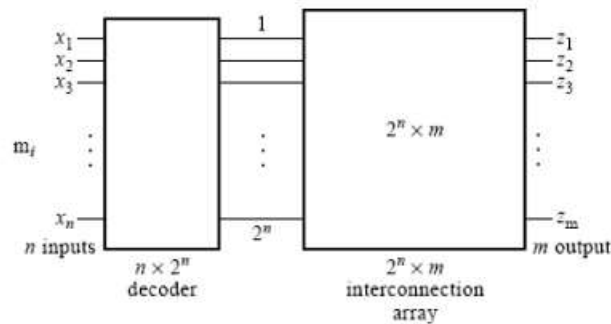
A ROM consists of two parts:

- An $n \times 2^n$ decoder
- A $2^n \times m$ array of switching devices that form interconnections between the 2^n lines from the decoder and the m output lines. The 2^n output lines from the decoder are called the word lines.

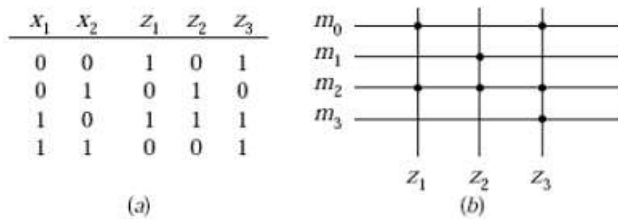
Each of the 2^n combinations that constitute the inputs to the interconnection array corresponds to a minterm and specifies an address. The memory consists of those connections that are actually made in the connection matrix between the word lines and the output lines. Once made, the connections in the memory array are permanent. So this memory is not one whose contents can be changed readily from time to time; we write into this memory but once. However, it is possible to read the information already stored (the connections actually made) as often as desired, by applying input words and observing the output words. That's why the circuit is called read-only memory. Before you continue reading, think of two possible ways in which to fabricate a ROM so

that one set of connections can be made and another set left unconnected. Continue reading after you have thought about it.

A ROM can be almost completely fabricated except that none of the connections are made. Such a ROM is said to be blank. Forming the connections for a particular application is called programming the ROM. In the process of programming the ROM, a mask is produced to cover those connections that are not to be made. For this reason, the blank form of the ROM is called mask programmable.



Basic structure of a ROM.



(a)

(b)

A ROM truth table and its program.

Programmable Logic Array

A programmable logic array is a kind of programmable logic devices (PLDs) used to implement combinational logic circuits. The PLA has a set of programmable OR gate planes, which can then be conditionally complemented to produce an output. It has 2^n AND gates for n input and for m outputs from PLA, there should be m OR gates. This layout allows for a larger number of logic functions to be synthesized in the sum of products canonical form.

PLA differs from programmable array logic devices in that both AND and OR gate planes are programmable.